Status of the standard vector—axial-vector model for nuclear beta decay

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The complete set of experimental results on correlations in nuclear beta decay is analyzed in terms of the general Hamiltonian including scalar, vector, axial vector, and tensor interactions with an arbitrary degree of parity violation. It is concluded that the standard vector minus axial-vector model with maximal parity violation (left-handed lepton current) is compatible with the data and rigorous limits are obtained for the values of possible additional coupling constants. In the scalar and tensor case the new constraints are considerably tighter than those published before: $|C_S/C_V|$ and $|C_S'/C_V|\leq 0.2$, $|(C_S+C_S')/C_V|\leq 0.06$; $|C_T/C_A|$ and $|C_T'/C_A|\leq 0.09$, $|(C_T+C_T')/C_A|\leq 0.01$, all at the 95% confidence level. On the other hand, rather large admixtures of the right-handed lepton currents ($C_V'/C_V\neq 1$ or $C_A'/C_A\neq 1$) are allowed by the data. An analysis of the correlations between various coupling constants implied by the data is also performed.

[RADIOACTIVITY Correlations in β decay analyzed; limits on unusual couplings deduced.]

I. INTRODUCTION

The standard vector minus axial-vector ($V-A$) model of the weak interactions, based on the sound principles of gauge invariance and renormalizability, is highly successful: it is consistent with the experimental data. However, one does expect that deviations from this model would be seen at some level. It is interesting to see to what degree the "exotic" couplings are excluded by experiments on low-energy semileptonic strangeness-conserving transitions (traditionally the most accurate type of weak interaction experiments). The most general such investigation to date was the least-squares adjustment of the beta-decay coupling constants performed by Paul[1] in 1970. That adjustment allowed substantial deviations from the standard $V-A$ model; the limits on the allowed levels of scalar and tensor interactions were particularly poor. Later coupling constant adjustments were less general, arbitrarily excluding scalar and tensor interactions[2,3] and sometimes also assuming maximal parity violation.[4,5]

This paper presents the results of least-squares adjustments similar to the adjustments of Paul's 1970 paper.[1] The emphasis, however, is on a more rigorous investigation of the limits on the coupling constants implied by the experimental data; in addition, the inclusion of data obtained since 1970 yields substantially tighter constraints on the coupling constants.

Only data on nuclear beta decay are considered in the present fit. No attempt is made to include purely leptonic processes, such as the muon decay, because there is no reason to expect that possible deviations from the standard $V-A$ model will be universal. Similarly, the possible indications for right-handed current effects in $\Delta S=1$ semileptonic decays[6] are not considered here.

II. ASSUMPTIONS: BETA DECAY THEORY

We consider only allowed transitions. The weak interaction Hamiltonian may then be written as[7]

$$H_{\rm int}=\frac{G_W}{\sqrt{2}}\sum_i(\bar\psi_{\rm p}O_i\psi_{\rm n})[\bar\psi_e O_i(C_i+C_i'\gamma_5)\psi_\nu]+{\rm H.c.}\ , \qquad (1)$$

where $i=S, V, T, A$, and

$$\begin{aligned}O_S&=1\ ,\\ O_V&=\gamma_\mu\ ,\\ O_T&=-\frac{i}{2\sqrt{2}}(\gamma_\mu\gamma_\lambda-\gamma_\lambda\gamma_\mu)\ ,\\ O_A&=-i\gamma_\mu\gamma_5\ .\end{aligned} \qquad (2)$$

(The pseudoscalar contribution vanishes to lowest order, since $O_P=\gamma_5$ couples large to small components of the nuclear wave functions and thus $\bar\psi_{\rm p}O_P\psi_{\rm n}$ is very small. Following Paul's lead, we neglect such "higher order" terms as weak magnetism, and corrections that the use of exact electron radial wave functions would produce.) In this paper we restrict ourselves to the case where the coupling constants C_i and C_i' are real: all time-reversal violating terms can then be neglected.

The following types of experimental measurements were available for use in the least-squares adjustments: a, the electron-neutrino angular correlation; b, the Fierz interference term; A, the electron angular distribution from polarized nuclei; B, the neutrino angular distribution from polarized nuclei; $\mathring{A}$, the electron-circularly polarized gamma ray-angular correlation; G, the electron helicity as a fraction of v/c; and $t_{\rm n}$, the half-life of the neutron.

Each of these measurable quantities can be expressed as a ratio of bilinear functions of the coupling constants (see the Appendix); except for the half-life of the neutron, they do not depend on the weak interaction coupling strength G_W. The formula for the neutron half-life t_n in terms of the coupling constants is rendered independent of G_W by expressing it in terms of the (much more accurately measured) ft value for Fermi $0^+ \to 0^+$ superallowed transitions (extrapolated to $Z=0$).[5] For pure Fermi or pure Gamow-Teller transitions, the formulas are independent of nuclear matrix elements.

III. LEAST-SQUARES METHOD AND ERROR HANDLING

Least-squares adjustments were performed for the following set of seven (real) parameters (and several subsets thereof):

$$p_1=\frac{C_A}{C_V},\; p_2=\frac{C_S}{C_V},\; p_3=\frac{C_T}{C_A},$$

$$p_4=\frac{C_V'}{C_V},\; p_5=\frac{C_A'}{C_A},\; p_6=\frac{C_S'}{C_V},$$

and

$$p_7=\frac{C_T'}{C_A}. \tag{3}$$

If f_i is the ith experimental measurement, having uncertainty σ_i, and $\phi_i(p_1,\ldots,p_n)$ is the corresponding theoretical function of the parameters (see the Appendix for definitions), then chi squared is defined as

$$\chi^2=\sum_{i=i}^{N}\frac{(f_i-\phi_i)^2}{\sigma_i^2}. \tag{4}$$

One wishes to find that set $\vec{p}^{\,(0)}$ of parameter values which minimizes χ^2. Defining $\vec{L}$ by

$$L_j\equiv\frac{1}{2}\frac{\partial\chi^2}{\partial p_j}=-\sum_{i=i}^{N}\frac{(f_i-\phi_i)}{\sigma_i^2}\frac{\partial\phi_i}{\partial p_j} \tag{5}$$

and the (symmetric) matrix $\underline{M}$ by

$$M_{jk}\equiv\frac{1}{2}\frac{\partial^2\chi^2}{\partial p_k\partial p_j}=\sum_{i=i}^{N}\frac{1}{\sigma_i^2}\frac{\partial\phi_i}{\partial p_k}\frac{\partial\phi_i}{\partial p_j}-\sum_{i=i}^{N}\frac{(f_i-\phi_i)}{\sigma_i^2}\frac{\partial^2\phi_i}{\partial p_k\partial p_j} \tag{6}$$

(where the second term of $\underline{M}$ is frequently omitted as being negligible), the standard method[8,9] for finding the position $\vec{p}^{\,(0)}$ of the minimum of chi squared (given some initial guess $\vec{p}^{\,(1)}$) involves iterating the equation

$$\vec{p}^{\,(n+1)}=\vec{p}^{\,(n)}-\vec{L}(\vec{p}^{\,(n)})\underline{M}^{-1}(\vec{p}^{\,(n)}), \tag{7}$$

which is based on a Taylor expansion (to quadratic order) of χ^2 about $\vec{p}^{\,(n)}$. If the quadratic Taylor expansion is not a good approximation (over an area including $\vec{p}^{\,(0)}$), Eq. (7) may not converge, or $\underline{M}$ may be singular (to the accuracy of its representation in computer memory). To surmount these difficulties, we used the Marquardt method[10]: this involves replacing $\underline{M}^{-1}$ in Eq. (7) by $\underline{M}'^{-1}$, where

$$M_{jk}'=M_{jk}+\delta_{jk}\lambda\,|M_{jj}|, \tag{8}$$

where δ_{jk} is the Kronecker delta and λ is a (small) positive number.

If the quadratic Taylor expansion of chi squared about the minimum $\vec{p}^{\,(0)}$ is good over a large enough region of parameter space, then $\underline{M}^{-1}$ (calculated at $\vec{p}^{\,(0)}$) is the covariance matrix of the parameters; one generally uses this to estimate the errors in the adjustment. For the more general fits, this approximation turned out to be poor: $\underline{M}^{-1}$ gave a very poor estimate of the limits on the parameters. Also, certain pairs of parameters turned out to be strongly correlated. To obtain confidence limits on the parameters, we looked instead at (hyper) surfaces of constant chi squared.

In the limit of a large number N of measurements, the surface defined by

$$\chi^2(\vec{p})=\chi^2(\vec{p}^{\,(0)})+k^2\equiv\chi_0^2+k^2 \tag{9}$$

encloses a confidence region in parameter space having the same confidence level (i.e., probability) as the region between $\pm k$ of a unit Gaussian: such a region might be loosely thought of as a "$k\sigma$" region. Projections (not cross-sectional slices) of such surfaces onto two-dimensional planes in parameter space were obtained for strongly correlated pairs of parameters; projections onto each parameter axis gave confidence limits for each parameter (though these limits are not independent, owing to correlations between parameters).

Provided that the experimental measurements obey Gaussian distribution rules, the least-squares method used here is equivalent to the maximum likelihood method as recommended by Annis *et al.*[11] The confidence region given by Eq. (9) defines corresponding confidence regions of the functions $\phi_i(\vec{p})$. Unlike the experimental measurements f_i, the functions ϕ_i are, in general, restricted to certain finite intervals (e.g., helicity is restricted to the interval ± 1). The confidence regions of the ϕ_i naturally respect these intervals. Our method of determining the boundaries of the confidence regions of the parameters p_j is equivalent to solving for p_j at the boundary of the confidence region of ϕ_i as done, e.g., in Ref. 3.

IV. SELECTION OF EXPERIMENTAL DATA

The experimental data values used in the least-squares adjustments are given in Table I, with their corresponding references. Of the 69 data values used by Paul[1] in his 1970 fits, all but one are included: the excluded measurement[12] (of the Fierz term: $b=0.0014\pm0.024$) is from the mixed (mirror) transition $^{13}N \to {}^{13}C$. In addition we include 26 more recent measurements on pure Fermi and pure Gamow-Teller transitions and on neutron decay. These 92 data values are the ones used to obtain the limits on the coupling constants presented in Table II: as mentioned above, the nuclear matrix elements do not enter into the formulas for pure transitions, and for neutron decay the matrix elements can be calculated easily and accu-

TABLE I. Experimental data values.

Measured quantity	Measurement type number	Decay	Value (error)	Electron energy W[a] Units of m_ec^2	Reference for data value	$\Delta\chi^2$ for most general fit (seven parameter)	$\Delta\chi^2$ for $V-A$ fit (one parameter)	Weighted average value (error)	Weighted average of electron energy
Å	1	${}^{22}_{11}Na \xrightarrow[GT]{\beta^+} {}^{22}_{10}Ne$	0.295(0.054)	1.347	1	0.49	0.50		
			0.368(0.028)[b]	1.371	2	1.59	1.53		
			0.332(0.028)[b]	1.561	2	0.00	0.00		
			0.36(0.08)	1.347	3	0.12	0.11		
								0.3443(0.0181)	1.4465
			0.334(0.017)	2.01	4	0.01	0.00		
								0.334(0.017)	2.01
	2	${}^{60}_{27}Co \xrightarrow[GT]{\beta^-} {}^{60}_{28}Ni$	−0.32(0.07)	1.25	5	0.03	0.04		
			−0.335(0.018)	1.217	1	0.01	0.01		
			−0.312(0.095)[b]	1.178	2	0.05	0.05		
			−0.247(0.067)[b]	1.217	2	1.65	1.66		
			−0.323(0.039)[b]	1.241	2	0.07	0.07		
			−0.203(0.138)	1.163	6	0.89	0.89		
			−0.299(0.016)[b]	1.243	7[c]	4.52	4.60		
								−0.3131(0.0110)	1.2312
			−0.39(0.05)	1.32	8	1.30	1.28		
			−0.352(0.032)[b]	1.275	2	0.35	0.34		
			−0.375(0.032)[b]	1.334	2	1.72	1.70		
								−0.3680(0.0206)	1.3071
			−0.40(0.09)	1.382	9	0.55	0.55		
			−0.393(0.041)[b]	1.4	2	2.14	2.12		
			−0.361(0.082)	1.382	6	0.12	0.11		
			−0.333(0.018)[b]	1.354	7	0.00	0.00		
			−0.322(0.014)	1.43	4	0.62	0.66		
								−0.3321(0.0105)	1.4007
			−0.36(0.06)	1.48	8	0.20	0.20		
			−0.291(0.027)[b]	1.459	2	2.42	2.46		
			−0.366(0.024)[b]	1.514	2	1.89	1.85		
			−0.361(0.022)[b]	1.55	7	1.62	1.58		
								−0.3449(0.0135)	1.5121
B	3	$n \xrightarrow[mixed]{\beta^-} p$	1.01(0.04)[d]	1.7[e]	10	0.33	0.30		
			0.96(0.40)	1.7[e]	11	0.00	0.00		
			0.995(0.034)[f]	1.7	12	0.06	0.04		

TABLE I. *(Continued)*.

Measured quantity	Measurement type number	Decay	Value (error)	Electron energy W[a] Units of m_ec^2	Reference for data value	$\Delta\chi^2$ for most general fit (seven parameter)	$\Delta\chi^2$ for $V-A$ fit (one parameter)	Weighted average value (error)	Weighted average of electron energy
								1.0011(0.0259)	1.7
A	4	$n \xrightarrow[\text{mixed}]{\beta^-} p$	−0.104(0.011)[f]	1.42	13	0.82	0.82		
			−0.115(0.012)[f]	1.373	14	0.01	0.01		
								−0.1090(0.0081)	1.3985
			−0.115(0.008)[d]	1.7[e]	10	0.02	0.02		
			−0.09(0.05)	1.9	15	0.23	0.23		
			−0.116(0.011)[f]	1.747	13	0.03	0.03		
			−0.118(0.010)[f]	1.7[e]	16	0.16	0.16		
			−0.115(0.007)[f]	1.722	14	0.02	0.02		
								−0.1155(0.0043)	1.7168
G	5	${}^{14}_{8}O \xrightarrow[\text{F}]{\beta^+} {}^{14}_{7}N$	0.97(0.19)	3.0	17	0.01	0.02		
								0.97(0.19)	3.0
	6	${}^{12}_{5}B \xrightarrow[\text{GT}]{\beta^-} {}^{12}_{6}C$	−0.98(0.06)	21.0	18	0.10	0.11		
								−0.98(0.06)	21.0
G	7	${}^{32}_{15}P \xrightarrow[\text{GT}]{\beta^-} {}^{32}_{16}S$	−1.05(0.04)	4.3	19	1.64	1.56		
								−1.05(0.04)	4.3
			−0.999(0.053)	2.937	20	0.00	0.00		
								−0.999(0.053)	2.937
			−1.025(0.035)	2.605	20	0.56	0.51		
								−1.025(0.035)	2.605
			−0.99(0.02)[g]	2.2	21	0.20	0.25		
			−0.983(0.029)	2.292	20	0.30	0.34		
								−0.9877(0.0165)	2.2297
			−1.01(0.03)	1.68	22	0.14	0.11		
			−1.02(0.03)	1.8	22	0.49	0.44		
			−1.00(0.03)	1.94	22	0.00	0.00		
								−1.0100(0.0173)	1.8067

TABLE I. *(Continued)*.

Measured quantity	Measurement type number	Decay	Value (error)	Electron energy W^a Units of m_ec^2	Reference for data value	$\Delta\chi^2$ for most general fit (seven parameter)	$\Delta\chi^2$ for $V-A$ fit (one parameter)	Weighted average value (error)	Weighted average of electron energy
			−1.00(0.04)	1.43	22	0.00	0.00		
			−1.00(0.04)	1.57	22	0.00	0.00		
			−0.97(0.03)[f]	1.511	23	0.93	1.00		
			−1.01(0.04)[f]	1.337	23	0.07	0.06		
								−0.9909(0.0183)	1.4700
G	8	${}^{60}_{27}\mathrm{Co} \xrightarrow[\mathrm{GT}]{\beta^-} {}^{60}_{28}\mathrm{Ni}$	−1.01(0.03)[f]	1.511	23	0.13	0.11		
								−1.01(0.03)	1.511
			−0.994(0.057)	1.38	24	0.01	0.01		
			−1.007(0.034)	1.4	25	0.05	0.04		
			−0.989(0.030)[f]	1.4	26	0.11	0.13		
			−0.99(0.03)[f]	1.399	23	0.09	0.11		
								−0.9944(0.0172)	1.3979
			−0.96(0.06)	1.32	27	0.42	0.44		
			−1.017(0.027)	1.25	25	0.44	0.40		
			−1.05(0.04)[f]	1.25	23	1.62	1.56		
			−0.96(0.03)[f]	1.315	23	1.70	1.78		
			−1.018(0.028)[f]	1.25	26	0.45	0.41		
								−1.0047(0.0146)	1.2697
			−1.081(0.081)	1.096	25	1.02	1.00		
			−1.047(0.047)	1.155	25	1.04	1.00		
			−0.72(0.12)[f]	1.091	23	5.41	5.44		
			−0.92(0.03)[f]	1.155	23	6.96	7.11		
			−1.056(0.048)	1.155	26	1.40	1.36		
			−1.061(0.093)[f]	1.096	26	0.44	0.43		
								−0.9814(0.0207)	1.1463
	9	${}^{68}_{31}\mathrm{Ga} \xrightarrow[\mathrm{GT}]{\beta^+} {}^{68}_{30}\mathrm{Zn}$	0.99(0.09)	3.3	20	0.01	0.01		
			0.829(0.093)[f,h]	2.4[e]	28	3.31	3.38		
								0.99(0.09)	3.3
	10	${}^{3}_{1}\mathrm{H} \xrightarrow[\mathrm{mixed}]{\beta^-} {}^{3}_{2}\mathrm{He}$	−1.005(0.026)[f,i]	1.0297	29	0.07[j]	0.04[j]		
								−1.005(0.026)	1.0297

TABLE I. *(Continued)*.

Measured quantity	Measurement type number	Decay	Value (error)	Electron energy W[a] Units of m_ec^2	Reference for data value	$\Delta\chi^2$ for most general fit (seven parameter)	$\Delta\chi^2$ for $V-A$ fit (one parameter)	Weighted average value (error)	Weighted average of electron energy
	11	${}^{26}_{13}\mathrm{Al}^{m} \xrightarrow[F]{\beta^+} {}^{26}_{12}\mathrm{Mg}$[k]	0.986(0.039)[f]	5.305	30	0.00	0.13		
								0.986(0.039)	5.305
b	12	${}^{22}_{11}\mathrm{Na} \xrightarrow[GT]{\beta^+} {}^{22}_{10}\mathrm{Ne}$[l]	0.031(0.037)		31	0.67	0.70		
			0.028(0.040)		32	0.47	0.49		
			0.036(0.035)		33	1.02	1.06		
			0.013(0.022)[m]		34	0.31	0.35		
			0.015(0.022)[m]		35	0.42	0.46		
			−0.016(0.020)		36	0.69	0.64		
			0.0008(0.0020)		37	0.00	0.16		
			0.2(0.1)		38	3.97	4.00		
			0.0008(0.0028)		39	0.00	0.08		
								0.0011(0.0016)	
	13	${}^{114}_{49}\mathrm{In} \xrightarrow[GT]{\beta^-} {}^{114}_{50}\mathrm{Sn}$	0.05(0.02)		40	6.41	6.25		
			0.005(0.022)		41	0.07	0.05		
								0.0296(0.0148)	
	14	${}^{38}_{19}\mathrm{K}^{m} \xrightarrow[F]{\beta^+} {}^{38}_{18}\mathrm{Ar}$	−0.03(0.08)[f]		42	0.15	0.14		
								−0.03(0.08)	
	15	Dependence on electron energy of ft values of Fermi $0^+ \rightarrow 0^+$ superallowed transitions	−0.001(0.006)[f]		43	0.00	0.03		
								−0.001(0.006)	
a	16	$\mathrm{n} \xrightarrow[mixed]{\beta^-} \mathrm{p}$	0.07(0.12)		44	2.06	2.06		
			−0.06(0.13)		45	0.11	0.11		
			−0.091(0.039)		46	0.08	0.08		
			−0.099(0.008)[f]		47	0.16	0.16		
								−0.0978(0.0078)	

TABLE I. (*Continued*).

Measured quantity	Measurement type number	Decay	Value (error)	Electron energy W^a Units of m_ec^2	Reference for data value	$\Delta\chi^2$ for most general fit (seven parameter)	$\Delta\chi^2$ for $V-A$ fit (one parameter)	Weighted average value (error)	Weighted average of electron energy
	17	${}^6_2\mathrm{He} \xrightarrow[\mathrm{GT}]{\beta^-} {}^6_3\mathrm{Li}$	$-0.39(0.05)$		48	1.29	1.28		
			$-0.33(0.01)$		49	0.11	0.11		
			$-0.35(0.04)$		50	0.17	0.17		
			$-0.3343(0.003)$		51	0.11	0.10		
			$-0.319(0.028)$		52	0.26	0.26		
								$-0.3341(0.0028)$	
	18	${}^{23}_{10}\mathrm{Ne} \xrightarrow[\mathrm{GT}]{\beta^-} {}^{23}_{11}\mathrm{Na}$	$-0.37(0.04)$		48	0.84	0.84		
			$-0.33(0.03)$		53	0.01	0.01		
								$-0.3444(0.0240)$	
	19	${}^{35}_{18}\mathrm{Ar} \xrightarrow[\mathrm{mixed}]{\beta^+} {}^{35}_{17}\mathrm{Cl}$	$0.9(0.3)^{f,i}$		54	0.03^j	0.03^j		
			$0.7(0.17)^{f,i}$		54	0.77^j	0.77^j		
			$0.97(0.14)^{f,i}$		48	0.74^j	0.74^j		
								0.8654(0.1017)	
t_n	20	$\mathrm{n} \xrightarrow[\mathrm{mixed}]{\beta^-} \mathrm{p}$	$636.6(9.6)^{f,i,n}$		55	2.90^j	2.75^j		
			$649.0(12.0)^{f,i,n}$		56	5.74^j	5.57^j		
			$607.8(5.4)^{f,i,o}$		57	5.32^j	5.69^j		
			$620.0(12.0)^{f,p}$		58	0.08	0.08		
								620.0(12.0)	

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TABLE I. *(Continued).*

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TABLE I. *(Continued)*.

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[a]Electron energy includes rest energy.
[b]Value read from curve (Paul's estimate used).
[c]Value of $\mathring{A}=-0.251(0.016)$ at $W=1.167$ from Ref. 7 excluded: $\Delta\chi^2\simeq 26$.
[d]Includes value from M. T. Burgy, V. E. Kronin, T. B. Novey, and G. R. Ringo, Phys. Rev. 120, 1829 (1960).
[e]Energy estimated.
[f]New data value: not used in Paul's 1970 fit.
[g]Systematic error used, since it is larger than quoted statistical error of 0.009.
[h]This value was inadvertently omitted from the averages used when obtaining contours; fortunately, its omission makes no significant difference to the limits.
[i]These values were not used in obtaining the limits of Table II.
[j]Values of $\Delta\chi^2$ from fit where *all* tabulated data values were used.
[k]Isomer at 0.229 MeV ($J^\pi=0^+$, $t_{1/2}=6.4$ sec): the authors actually measured the ratio G_F/G_{GT} using $^{26}Al^m$ and ^{30}P, but since $|G_{GT}|=1$ to an accuracy of better than 1% even if this measurement is excluded from the fit, the quoted error of 0.038 in the ratio is only increased to 0.039 for G_F itself.
[l]Two values of b excluded from fit owing to excessive $\Delta\chi^2$: $b=0.30(0.05)$ from J. H. Hamilton, L. M. Langer, and W. G. Smith, Phys. Rev. 112, 2010 (1958): $\Delta\chi^2\simeq 36$; $b=0.25(0.05)$ from O. E. Johnson, F. Pleasonton, and L. M. Langer, Phys. Rev. 112, 2004 (1958): $\Delta\chi^2\simeq 25$.
[m]Electron exchange correction included (by Paul, who took value of correction from Ref. 38).
[n]High neutron half-life values (in sec).
[o]Low neutron half-life value (in sec).
[p]This mean neutron half-life value was obtained by Wilkinson (see Ref. 58) by taking a weighted average of the above three half-life values with errors inflated (proportionately) so that the weighted average had chi-square per degree of freedom equal to one.

TABLE II. Limits on parameter values.

Case	Surface of constant χ^2 giving rise to parameter limit	Limits on parameter values C_A/C_V	C_S/C_V	C_T/C_A	C_V'/C_V	C_A'/C_A	C_S'/C_V	C_T'/C_A
I[a]	$\chi^2=\chi_0^2+3^2$	-0.962	0.328	0.118	1.674	1.218	0.249	0.106
	$\chi^2=\chi_0^2+2^2$	-1.000	0.240	0.093	1.531	1.178	0.190	0.085
	$\chi^2=\chi_0^2+1^2$	-1.051	0.142	0.062	1.377	1.128	0.118	0.057
	min A: $\chi^2=\chi_0^2$	-1.137	-0.006	0.004	0.848	1.055	0.006	-0.005
	min B: $\chi^2=\chi_0^2$	-1.415	0.008	-0.005	1.179	0.948	-0.007	0.004
	$\chi^2=\chi_0^2+1^2$	-1.579	-0.131	-0.058	0.726	0.886	-0.117	-0.058
	$\chi^2=\chi_0^2+2^2$	-1.712	-0.228	-0.090	0.653	0.849	-0.190	-0.086
	$\chi^2=\chi_0^2+3^2$	-1.838	-0.312	-0.114	0.597	0.821	-0.249	-0.107
II	3σ: $\chi^2=\chi_0^2+3^2$	-1.2374	0.0084	0.0021				
	2σ: $\chi^2=\chi_0^2+2^2$	-1.2451	0.0054	0.0013				
	1σ: $\chi^2=\chi_0^2+1^2$	-1.2528	0.0025	0.0005				
	min: $\chi^2=\chi_0^2$	-1.2606	-0.0004	-0.0003	1	1	$\equiv C_S/C_V$	$\equiv C_T/C_A$
	1σ: $\chi^2=\chi_0^2+1^2$	-1.2685	-0.0034	-0.0011				
	2σ: $\chi^2=\chi_0^2+2^2$	-1.2765	-0.0063	-0.0019				
	3σ: $\chi^2=\chi_0^2+3^2$	-1.2846	-0.0093	-0.0027				
III[a]	$\chi^2=\chi_0^2+3^2$	-0.965			1.671	1.213		
	$\chi^2=\chi_0^2+2^2$	-1.003			1.528	1.172		
	$\chi^2=\chi_0^2+1^2$	-1.054			1.374	1.122		
	minA': $\chi^2=\chi_0^2$	-1.142	0	0	0.850	1.049	0	0
	minB': $\chi^2=\chi_0^2$	-1.408	0	0	1.176	0.954	0	0
	$\chi^2=\chi_0^2+1^2$	-1.573			0.728	0.891		
	$\chi^2=\chi_0^2+2^2$	-1.707			0.655	0.853		
	$\chi^2=\chi_0^2+3^2$	-1.833			0.599	0.825		
IV	3σ: $\chi^2=\chi_0^2+3^2$	-1.2375						
	2σ: $\chi^2=\chi_0^2+2^2$	-1.2451						
	1σ: $\chi^2=\chi_0^2+1^2$	-1.2529						
	min: $\chi^2=\chi_0^2$	-1.2607	0	0	1	1	0	0
	1σ: $\chi^2=\chi_0^2+1^2$	-1.2686						
	2σ: $\chi^2=\chi_0^2+2^2$	-1.2766						
	3σ: $\chi^2=\chi_0^2+3^2$	-1.2847						

[a]An extra decimal place (not really significant) is given here so as not to distort the size of the differences between various limits.

rately (namely $|M_F|^2=1$, $|M_{GT}|^2=3$). For mixed transitions, the formulas for the theoretical functions ϕ_i do depend on the ratio of the matrix elements $R_M \equiv M_{GT}/M_F$, or at least on $|R_M|^2$. For certain superallowed mixed mirror transitions, having $|M_F|^2=1$, Raman *et al.*[13] give values for $|M_{GT}|^2$ obtained by comparing the ft values of the mirror transitions with the ft value for Fermi $0^+ \rightarrow 0^+$ superallowed transitions. However, the value obtained for $|M_{GT}|^2$ depends on the value one assumes for the coupling constants, so that for these transitions R_M should really be considered as another free parameter in the fit, its value being constrained by the transition's ft value. For the sake of simplicity, this was not done; when mixed transitions were used in the fit, their matrix element ratio was assumed to be fixed. (Because this procedure is not strictly valid, the limits of Table II were obtained without use of mixed transitions other than neutron decay.)

The above mentioned ^{13}N measurement is not sufficiently precise to affect the fit; the same is true of the measurement[14] of the beta angular distribution ($A=0.16\pm0.04$) from polarized ^{35}Ar, which would also require knowledge of the sign of R_M. Measurements [$A=-0.033\pm0.002$ (Ref. 14) and $A=-0.0391\pm0.0014$ (Ref. 15)] for ^{19}Ne are precise; but here there are electromagnetic corrections[15] of the same magnitude as the errors. Therefore, these data were not used in our fit. Fits were made using a helicity measurement of ^3H (see Table I); this tightened the limits on C_A/C_V, C_V'/C_V, and C_A'/C_A by a few percent. Two choices of matrix element ratios were tried[13]: $|R_M|^2=2.74$ and $|R_M|^2=2.92$. Fits were also made using measurements of a for ^{35}Ar (see Table I); this tightened limits on C_S/C_V and C_S'/C_V by less than ten percent. Again, two choices of matrix element ratios were tried[13]: $|R_M|^2=0.05$ and $|R_M|^2=0.08$. For both ^3H and ^{35}Ar, the different choices of matrix element ratios had little effect on the limits obtained.

As pointed out by Wilkinson,[5] the three modern neutron half-life values are not mutually consistent: two are high[16,17] and one is low[18] when compared to the half-life value predicted by the standard value of C_A/C_V. Wilkinson suggests using a weighted average with errors inflated to render the measurements consistent.[5] This value was used for the adjustments presented in this paper, but the effects of using other half-life values are discussed in Sec. V.

The minima and single-parameter limits of Tables II and III were obtained using the 92 measurements on pure transitions and neutron decay; the two-dimensional projections of the figures were obtained using 33 weighted averages (also given in Table I) of the measurements, including the two mixed decays. It was tested that the averaging had no significant effect on the limits; nor did use of the mixed transition data, as mentioned above.

The errors σ_i used in the adjustments were taken to be the uncertainties quoted by those who made the measurements. They included both statistical and systematic errors (added in quadrature).

V. RESULTS

Four main types of fits were performed, essentially in decreasing order of generality; case I (the full seven-parameter fit), case II (a "left-handed" three-parameter fit, constrained by the requirement $C_i' \equiv C_i$), case III (the general three-parameter fit containing only vector and axial vector couplings, with scalar and tensor couplings constrained to be zero), and case IV (the standard $V-A$ one-parameter fit with C_A/C_V being the only free parameter). The extreme allowed limits for each parameter (and their values at the minimum) are given for each of the four cases in Table II; they were obtained using the mean value for the neutron lifetime (with its error expanded as per Wilkinson[5]). The values χ_0^2 of chi squared at the minimum are given in Table III for the four different possible choices of neutron lifetime (Wilkinson's average,[5] all three values,[16–18] high values only,[16,17] or low value only[18]). Also presented in Table III are the corresponding values and errors obtained for C_A/C_V in the fit of case IV for each choice of neutron lifetime.

In every case, the value χ_0^2 of chi squared at the minimum is not significantly different (at the 90% confidence level) from the number n_D of degrees of freedom. Thus, unlike Paul[1] in 1970, we did not normalize χ_0^2 to n_D: the limits on the parameters were defined by $\chi^2=\chi_0^2+k^2$ not

$$(n_D/\chi_0^2)\chi^2=(n_D/\chi_0^2)\chi_0^2+k^2 \; .$$

(If the latter definition had been used, the limits on the parameter values would have been tightened by less than ten percent.)

Case I: Full seven-parameter fit. There were two minima (labeled A and B in tables and figures): two different sets of parameter values yield exactly the same minimum value for chi squared, owing to the fact that they yield exactly the same theoretical estimates ϕ_i of the experimental data values f_i. However, the surface $\chi^2=\chi_0^2+1$ encloses both minima as well as the "$V-A$ position"; by looking at the value of χ_0^2 for case IV, one sees that the "ridge" between the two minima rises only to about $\chi_0^2+0.35$.

TABLE III. Chi-square values for various types of fits.

			Neutron half-lives used in fits			
Case	Description of fit	Quantity quoted in body of table	Wilkinson's mean[a] neutron half-life	All three neutron half-lives	Weighted average of high[b,c] neutron half-lives	Low[d] neutron half-life
I	full seven-parameter fit	χ^2 at minimum $\equiv\chi_0^2$	72.59	86.69	72.53	74.81
		degrees of freedom n_D	85	87	85	85
II	left-handed three-parameter fit	χ^2 at minimum $\equiv\chi_0^2$	72.77	86.97	75.03	76.02
		degrees of freedom n_D	89	91	89	89
III	no scalar or tensor: three-parameter fit	χ^2 at minimum $\equiv\chi_0^2$	72.79	86.90	73.36	75.09
		degrees of freedom n_D	89	91	89	89
IV	$V-A$ single-parameter fit	χ^2 minimum $\equiv\chi_0^2$	72.94	87.13	75.23	76.16
		degrees of freedom n_D	91	93	91	91
		$V-A$ value of C_A/C_V	−1.2607	−1.2641	−1.2488	−1.2730
		error implied in fit	±0.0079	±0.0047	±0.0064	±0.0055

[a]D. H. Wilkinson, Nucl. Phys. A377, 474 (1982).
[b]C. J. Christensen *et al.*, Phys. Rev. D 5, 1628 (1972).
[c]J. Byrne *et al.*, Phys. Lett. 92B, 274 (1980).
[d]L. N. Bondarenko *et al.*, Pis'ma Zh. Eksp. Teor. Fiz. 28, 328 (1978) [JETP Lett. 28, 303 (1978)].

C_S and C_S' are strongly correlated, as may be seen in Fig. 1 (note the different scales on the two axes). Figure 2 demonstrates that C_T is even more strongly correlated with C_T'. For both scalar and tensor couplings, the sum C_i+C_i' is much more tightly constrained than the difference C_i-C_i', owing to the way these parameters enter into the formula for b, the Fierz interference term (see the Appendix). For these parameters, the two minima essentially coincide, and are not far from the $V-A$ position $C_S=C_S'=C_T=C_T'=0$. C_A/C_V and C_V'/C_V are quite strongly correlated (see Fig. 3), and C_A'/C_A is correlated to a lesser degree with each of them. For these three parameters, the two minima are fairly widely separated. (Case III, with no scalar or tensor allowed, yields almost identical results.)

The only significant effect (besides increasing χ_0^2 by 14) of using the three neutron half-lives separately with their quoted errors is to tighten the limits on C_S and C_S' by approximately 15%. Using only the low neutron half-life[18] increases χ_0^2 by 2.3 and increases the height of the ridge between the minima to $\chi_0^2+1.3$, while tightening the limits on C_S and C_S' by about 30%. Using only the high neutron half-lives[16,17] spreads the two minima apart (particularly for C_S and C_S') and increases the height of the ridge between the minima to $\chi_0^2+2.7$: the $V-A$ position is barely within the 90% confidence region of the parameters. The limits on C_S and C_S' are then loosened by about 30%.

Our analysis cannot decide which of the two conflicting neutron lifetimes is more likely to be correct. However, the mean neutron lifetime, as recommended by Wilkinson,[5] seems to be more consistent with all the other correlation data than either of the measured lifetimes.

Case II: Left-handed three-parameter fit. There is only a single minimum. There are very tight limits on C_S and C_T, owing to the Fierz interference term (by definition, these limits may be obtained by taking the intercepts of the contours of Figs. 1 and 2 on their vertical axes, and dividing by two). The limit on C_A/C_V is also very tight, being essentially the same as in case IV, the $V-A$ fit. Use of the three separate neutron half-life values tightens the error on C_A/C_V by about 40% and shifts its value somewhat; use of either the high or the low values tightens the error on C_A/C_V by about 25% and shifts its value by about two standard deviations (see Table III for case IV).

Case III: Three-parameter fit; no scalar or tensor. There are two minima, as in case I; in fact, the limits on C_A/C_V, C_V'/C_V, and C_A'/C_A are essentially the same as in case I, and Fig. 3 is applicable here too. The different choices of neutron half-life values have no significant ef-

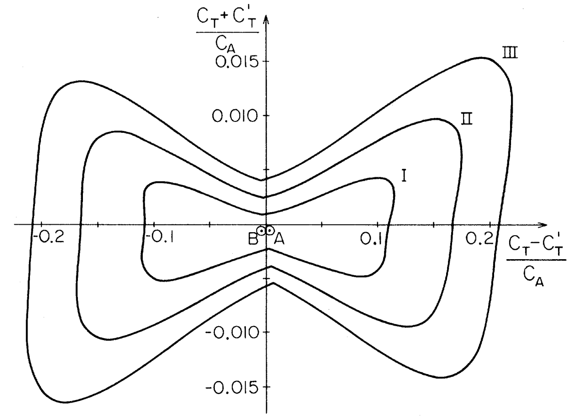

FIG. 2. Confidence regions for tensor coupling constants (general fit: case I), expressed in terms of the sum and difference of C_T/C_A and C_T'/C_A. For notation see Fig. 1. Note the different scales on the two axes.

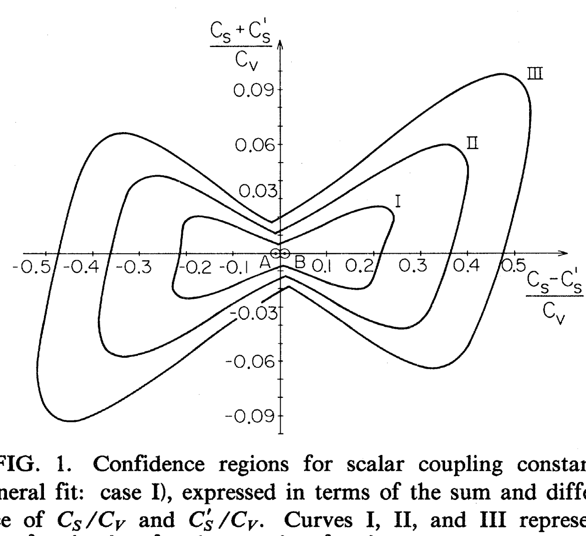

FIG. 1. Confidence regions for scalar coupling constants (general fit: case I), expressed in terms of the sum and difference of C_S/C_V and C_S'/C_V. Curves I, II, and III represent $\chi^2=\chi_0^2+1^2$, $\chi^2=\chi_0^2+2^2$, and $\chi^2=\chi_0^2+3^2$, respectively. The two minima of χ^2 are indicated by points A and B. Note the different scales on the two axes.

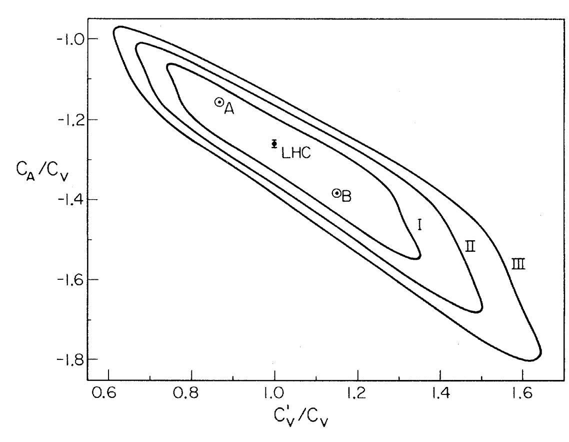

FIG. 3. Confidence regions showing the correlation of C_A/C_V with C_V'/C_V (general fit: case I; also applicable for case III). For notation see Fig. 1. The LHC point indicates the value and error of C_A/C_V for the left-handed fits (cases II and IV, where $C_i'\equiv C_i$).

fect on the limits.

Case IV: Standard $V-A$ one-parameter fit. The value and error for C_A/C_V obtained here are essentially the same as those obtained by Wilkinson[5] in his (restricted) 1982 fit. All comments about C_A/C_V in case II apply here too: see Table III for the effects of various choices of neutron half-life.

VI. CONCLUSIONS

For the most general seven-parameter fit, the limits on the parameters are the least stringent, although the limits on scalar and tensor coupling constants are an order of magnitude tighter than those obtained by Paul[1] in 1970. Looking at Table II and Figs. 1 and 2, one obtains the following 95% confidence limits (CL). For the scalar coupling constants, one finds

$$\left|\frac{C_S}{C_V}\right| < 0.23, \quad \left|\frac{C_S'}{C_V}\right| < 0.19: \ 95\% \text{ CL [mostly owing to } a(\text{n}) \text{ and } t_{\text{n}}] , \tag{10}$$

with

$$\left|\frac{C_S+C_S'}{C_V}\right| < 0.065: \ 95\% \text{ CL (mostly owing to } b_{\text{Fermi}}) . \tag{11}$$

For the tensor coupling constants,

$$\left|\frac{C_T}{C_A}\right| < 0.09, \quad \left|\frac{C_T'}{C_A}\right| < 0.085: \ 95\% \text{ CL [mostly owing to } a(^6\text{He})] , \tag{12}$$

with

$$\left|\frac{C_T+C_T'}{C_A}\right| < 0.01: \ 95\% \text{ CL [mostly owing to } b(^{22}\text{Na})] . \tag{13}$$

For the other three coupling constants, the limits obtained here are not all that much tighter than the limits obtained in the restricted fit of van Klinken *et al.*[3] in 1978: our limits are

$$-1.71 < \frac{C_A}{C_V} < -1.00: \ 95\% \text{ CL (many measurements contribute to this limit)} , \tag{14}$$

$$0.65 < \frac{C_V'}{C_V} < 1.53: \ 95\% \text{ CL [mostly owing to } G(^{26}\text{Al}^m)] , \tag{15}$$

$$0.85 < \frac{C_A'}{C_A} < 1.18: \ 95\% \text{ CL [mostly owing to } G(^{32}\text{P and } ^{60}\text{Co})] , \tag{16}$$

where it must be remembered that these latter three parameters are fairly strongly correlated with each other, especially C_A/C_V with C_V'/C_V. These confidence limits, and the confidence regions shown in the figures, are certainly consistent with the $V-A$ values for the coupling constants of

$$\frac{C_A}{C_V} = -1.26 , \qquad \frac{C_V'}{C_V} = \frac{C_A'}{C_A} = 1 , \tag{17}$$

and

$$C_S = C_S' = C_T = C_T' = 0 ,$$

Further, the $V-A$ values lie near the center of the above confidence regions, although the confidence regions are too irregular to allow one to quote a central value plus or minus an error. Note that constraining the scalar and tensor coupling constants to vanish has practically no effect on the other coupling constants.

For the case where C_i' is constrained to be equal to C_i (left-handed fit), the limits are tight, and it is possible to represent the coupling constants as values with errors:

$$\frac{C_A}{C_V} = -1.2607 \pm 0.0079 \ \text{[mostly owing to } A(\text{n}) \text{ and } t_{\text{n}}] , \tag{18}$$

$$\frac{C_S}{C_V} = -0.0004 \pm 0.0029 \ \text{[mostly owing to } b_{\text{Fermi}}] , \tag{19}$$

$$\frac{C_T}{C_A} = -0.0003 \pm 0.0008 \ \text{[mostly owing to } b(^{22}\text{Na})] . \tag{20}$$

(The 95% CL's are of course obtained by doubling the above error values.) Again, it makes no difference to C_A/C_V if the scalar and tensor coupling constants are constrained to vanish.

It is desirable to have a reliable value (and error) for the neutron half-life, which has some effect on the scalar limits and strongly affects the value and error of C_A/C_V in the standard $V-A$ fit (as may be seen from Table III). Certain proposed highly accurate measurements of ratios of positron polarizations[19,20] could lead to limits on C'_V/C_V (and probably C_A/C_V) comparable to the limits on C'_A/C_A, although these coupling constants appear only quadratically in the helicity G. Prospects for tighter limits on C_S, C'_S, C_T, and C'_T in the general case are not too hopeful, as they appear only quadratically in the constraining measurements (except for the Fierz term b, which only constrains the sums $C_S+C'_S$ and $C_T+C'_T$). Use of measurements on mixed mirror transitions (with the matrix element ratios properly considered as free parameters constrained by ft values) might help to tighten some limits.

When more accurate experimental data are available it will also be necessary to reconsider whether the deviations from the allowed approximations are indeed negligible (as we have assumed in this paper).

APPENDIX

The dependence of the measureable parameters a, b, A, B, $\mathring{A}$, G, and t_n (multiplied by ξ, where ξ is defined below) on the coupling constants is given in Eqs. (A3)–(A9) below, in the same notation as that used by Paul.[1] The assumptions involved are mentioned in Sec. II. Note that all the fits performed for this paper assumed that the coupling constants be real. For conciseness, one writes

$$K_{ij} \equiv C_i C_j^* + C'_i C_j'^* = C_i C_j + C'_i C'_j \ , \tag{A1}$$

$$L_{ij} \equiv C'_i C_j^* + C_i C_j'^* = C'_i C_j + C_i C'_j \ . \tag{A2}$$

Then (as given by Lundby[7]) the electron-neutrino angular correlation is given by

$$a\xi = \tfrac{1}{3}\,|M_{\rm GT}|^2(K_{TT}-K_{AA}) + |M_{\rm F}|^2(K_{VV}-K_{SS}) \ , \tag{A3}$$

the Fierz interference term is given by

$$b\xi = \pm 2\gamma(\,|M_{\rm GT}|^2{\rm Re}K_{TA} + |M_{\rm F}|^2{\rm Re}K_{SV}) \ , \tag{A4}$$

the electron angular distribution from polarized nuclei is given by

$$A\xi = \left[\pm\lambda_{JJ'}\,|M_{\rm GT}|^2(L_{TT}-L_{AA}) + 2\delta_{JJ'}\left[\frac{J}{J+1}\right]^{1/2} M_{\rm F}M_{\rm GT}{\rm Re}(L_{ST}-L_{VA})\right] \Big/ (1+b/W) \ , \tag{A5}$$

the neutrino angular distribution from polarized nuclei is given by

$$B\xi = \left\{ \lambda_{JJ'}\,|M_{\rm GT}|^2 \left[\pm(L_{TT}+L_{AA}) + \frac{2\gamma}{W}{\rm Re}L_{TA}\right] - 2\delta_{JJ'}\left[\frac{J}{J+1}\right]^{1/2} M_{\rm F}M_{\rm GT} \times \left[{\rm Re}(L_{ST}+L_{VA}) \pm \frac{\gamma}{W}{\rm Re}(L_{SA}+L_{VT})\right]\right\} \Big/ (1+b/W) \ , \tag{A6}$$

the electron circularly polarized gamma-angular correlation (for a gamma transition of pure multipolarity L) is given by

$$\mathring{A}\xi = \left[\pm\mu_{JJ'}\,|M_{\rm GT}|^2(L_{TT}-L_{AA}) + 2\delta_{JJ'}\left[\frac{J}{J+1}\right]^{1/2} M_{\rm F}M_{\rm GT}{\rm Re}(L_{ST}-L_{VA})\right] \Big/ [(1+L)(1+b/W)] \ , \tag{A7}$$

and the electron helicity (divided by v/c) is given by

$$G\xi = \pm[\,|M_{\rm GT}|^2(L_{TT}-L_{AA}) + |M_{\rm F}|^2(L_{SS}-L_{VV})]/(1+b/W) \ . \tag{A8}$$

The neutron half-life is given by

$$t_n\xi = 2\left[\frac{(ft)_{0\to 0}^{z\to 0}}{f}\right] |M_{\rm F}|^2(K_{SS}+K_{VV}) \ , \tag{A9}$$

where Wilkinson[5] gives

$$(ft)_{0\to 0}^{z\to 0} = 3083.1 \pm 1.4 \text{ sec} \ ,$$
$$f = 1.714\,65 \pm 0.000\,15 \ , \tag{A10}$$

so that for the purposes of fitting one can take, as a constant,

$$2\left[\frac{(ft)_{0\to 0}^{z\to 0}}{f}\right] = 3596.2 \text{ sec} \ . \tag{A11}$$

Also, in the above equations

$$\xi = |M_{\rm GT}|^2(K_{TT}+K_{AA}) + |M_{\rm F}|^2(K_{SS}+K_{VV}) \ , \tag{A12}$$

$\gamma = \sqrt{1-(\alpha Z)^2}$, where $\alpha = \frac{1}{137}$ is the fine structure constant and Z is the atomic number of the daughter nucleus,

$$\lambda_{JJ'} = \begin{cases} 1 \\ 1/(J+1) \\ -J/(J+1) \end{cases} , \qquad \mu_{JJ'} = \begin{cases} 1 \\ -1/J \\ -(J+2)/(J+1) \end{cases} \quad \text{for} \quad \begin{cases} J\to J'=J-1 \\ J\to J'=J \\ J\to J'=J+1 \end{cases} , \tag{A13}$$

L is the multipolarity of the gamma ray and W is the electron (or positron) energy in units of $m_e c^2$. The upper (lower) sign in the above equations refers to electron (positron) emission, M_F and M_{GT} are the Fermi and Gamow-Teller matrix elements, respectively, and J (J') is the initial (final) spin quantum number of the decaying nucleus.

The terms $(1+b/W)$ in the denominators of Eqs. (A5)—(A8) arise from the fact that b is assumed to be zero when one analyzes experimental data to obtain the correlation parameters: this is discussed at some length by Paul.[1] The electron energy values W need not be given very precisely, since b is always very small and thus a large error in W still yields only a very small error in the corresponding correlation value. Note that for a pure Fermi or pure Gamow-Teller transition, the matrix element cancels out when one divides Eqs. (A3)—(A9) by ξ [Eq. (A12)] to get the formulas for the actual measurements.

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